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NEW IDEAS IN CHILD AND EDUCATIONAL PSYCHOLOGY

On the Subject of Teaching Mathematics in Primary School: Experimental Learning Experience¤

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ABSTRACT

Background. The article explores the psychological principles underlying the educational curriculum and teaching methods using a primary school mathematics course as an example.

Objective. The study aims to conduct a pedagogical and psychological analysis of Ya. Abramson's mathematics program, which is grounded in P. Galperin's theory of the gradual formation of mental actions and concepts.

Design. Based on the L. Vygotsky's ideas on the relationship between learning and mental development, as well P. Galperin theory of the gradual formation of mental actions and concepts, a scientific and psychological rationale is provided for Ya. Abramson's mathematics program for schoolchildren. A brief description of the program is given, followed by a psychological analysis of the testing results in Moscow schools. This program has been shown to be highly effective.

Results. The obtained data provide practical confirmation of the effectiveness of training based on the *third type of orientation and learning* according to P. Galperin's theory.

Conclusion. The results clearly support P. Galperin's position on the importance of emphasizing the subject matter of the area being studied from the earliest stages of school education. This, in turn, demonstrates the heuristic potential of P. Galperin's general psychological theory and its applicability to various areas of social interaction, particularly in addressing educational challenges.

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Keywords*:* Activity theory of learning; action; teaching; concept formation; formative method; mental development; educational psychology; didactics; education

Highlights:

- L. Vygotsky's theories concerning learning and mental development and P. Galperin's theory of the gradual formation of mental actions provides a scientific framework for examining the implications on psychological development in the context of curriculum design and teaching methodology.
- The development of Ya. Abramson's mathematics program for primary school informed by P. Galperin's theory, demonstrated high efficacy in development based on the third type of orientation and learning. This approach contributes to the formation of creative mathematical thinking and strengths cognitive interest.
- Experimental training confirmed, P. Galperin's thesis regarding the connection between children's intellectual capabilities and types of learning: first-graders are capable of mastering basic mathematical concepts.
- The analysis of the program confirms P. Galperin's position on the importance of introducing and emphasizing academic subjectmatter from the very first stages of learning.

АННОТАЦИЯ

Актуальность. В статье обсуждается проблема психологических оснований содержания и методов школьного обучения на примере курса математики начальной школы.

Цель исследования. Педагогический и психологический анализ авторской программы по математике Я.И. Абрамсона, которая базируется на теории поэтапного формирования умственных действий и понятий П.Я. Гальперина.

Процедура. С опорой на представления Л.С. Выготского о соотношении обучения и умственного развития и теорию поэтапного формирования умственных действий и понятий П.Я. Гальперина, дается научно-психологическое обоснование авторской программы по математике Я.И. Абрамсона для младших школьников, приводится ее краткое описание с последующим психологическим анализом результатов апробации в школах Москвы. Показана высокая эффективность данной программы.

Результаты. Полученные данные выступают практическим подтверждением результативности обучения, построенного по третьему типу ориентировки и учения в теории П.Я. Гальперина.

Выводы. Результаты наглядно демонстрируют верность положения П.Я. Гальперина о необходимости выделения предмета изучаемой области с первых шагов школьного обучения, что, в свою очередь, лишний раз иллюстрирует эвристический потенциал общепсихологической теории П.Я. Гальперина и возможности ее приложения к различным областям социальной практики, в первую очередь при решении образовательных проблем.

Ключевые слова*:* Деятельностная теория учения; действие; учение; формирование понятий; формирующий метод; умственное развитие; педагогическая психология; дидактика; образование

Ключевые положения:

- Представления Л.С. Выготского о соотношении обучения и умственного развития и теория поэтапного формирования умственных действий и понятий П.Я. Гальперина выступают научным основанием для разработки психологических вопросов содержания и методов школьного обучения

- Авторская программа по математике для начальной школы Я.И. Абрамсона, которая построена на теории П.Я. Гальперина, показала высокую эффективность обучения по третьему типу ориентировки и учения; такое обучение способствует формированию творческого математического мышления и укреплению познавательного интереса
- В ходе экспериментального обучения получил подтверждение тезис П.Я. Гальперина о связи интеллектуальных возможностей детей и типов учения: первоклассники оказываются в состоянии освоить основные математические понятия
- Анализ программы подтверждает положение П.Я.Гальперина о необходимости выделения учебного предмета с самых первых шагов его изучения

RESUMEN

Relevancia. El artículo aborda el problema de los fundamentos psicológicos de los contenidos y los métodos de la enseñanza escolar, tomando como ejemplo el curso de matemáticas en la escuela primaria.

Objetivo. Análisis pedagógico y psicológico del programa de matemáticas del autor Y.I. Abramson, que se basa en la teoría de la formación por etapas de las acciones y los conceptos mentales de P.Y. Galperin.

Diseño. Apoyándose en las ideas de L.S. Vygotsky sobre la relación entre aprendizaje y desarrollo mental y en la teoría de la formación por etapas de las acciones y los conceptos mentales de P.Y. Galperin, se proporciona una justificación científico-psicológica del programa de matemáticas del autor Y.I. Abramson para alumnos de primaria. Se presenta una breve descripción del programa, seguida de un análisis psicológico de los resultados de su aplicación experimental en escuelas de Moscú. Se muestra la alta eficacia de este programa.

Resultados. Los datos obtenidos constituyen una confirmación práctica de la eficacia del aprendizaje construido según el tercer tipo de orientación y aprendizaje en la teoría de P.Ya. Galperin.

Conclusión. Los resultados demuestran claramente la validez de la afirmación de P.Ya. Galperin sobre la necesidad de identificar el objeto del área de estudio desde los primeros pasos de la enseñanza escolar, lo que, a su vez, ilustra una vez más el potencial heurístico de la teoría psicológica general de P.Ya. Galperin y las posibilidades de su aplicación a diversas áreas de la práctica social, principalmente en la resolución de problemas educativos.

Palabras clave: Teoría de la actividad del aprendizaje; acción; aprendizaje; formación de conceptos; método formativo; desarrollo mental; psicología pedagógica; didáctica; educación

Disposiciones clave:

- Las ideas de L.S. Vygotsky, sobre la relación entre aprendizaje y desarrollo mental, y la teoría de la formación por etapas de las acciones mentales y los conceptos de P. Y. Galperin, sirven como base científica para el desarrollo de cuestiones psicológicas de los contenidos y los métodos de la enseñanza escolar
- El programa de matemáticas de Y.I. Abramson para la escuela primaria, construido sobre la teoría de P.Y. Galperin, mostró una alta eficacia en el aprendizaje según el tercer tipo de orientación y aprendizaje; tal aprendizaje favorece la formación del pensamiento matemático creativo y el fortalecimiento del interés cognitivo
- En el curso del aprendizaje experimental se confirmó la tesis de P.Y. Galperin sobre la relación entre las capacidades intelectuales de los niños y los tipos de aprendiza-

je: los alumnos de primer grado son capaces de dominar conceptos matemáticos básicos

- El análisis del programa confirma la afirmación de P.Y. Galperin sobre la necesidad de identificar el objeto de la materia de estudio desde los primeros pasos de su aprendizaje

RESUME

Origines. Cet article aborde sur le problème du fondement psychologique du contenu et des méthodes de l`enseignement scolaire à partir de l`exemple d`un cours de mathématique à l`école primaire.

Objectif. Analyse pédagogique et psychologique du programme de mathématiques de l'auteur Yakov Abramson, qui est basé sur la théorie de la formation progressive d'actions et de concepts mentaux de P.Ya. Galpérine.

Méthodes. Basé sur les idées de Lev Vygotski sur la relation entre l'apprentissage et le développement mental et la théorie de la formation progressive des actions et des concepts mentaux par P. Ya. Galpérine, une base scientifique et psychologique est donnée au programme de mathématiques de l'auteur par Yakov Abramson pour les écoliers plus jeunes, une brève description est donnée, suivie d'une analyse psychologique des résultats des tests dans les écoles de Moscou. L`article a démontré une haute efficacité de ce programme.

Résultats. Les données obtenues servent de confirmation pratique de l'efficacité de la formation basée sur le troisième type d'orientation et d'apprentissage dans la théorie de P. Ya. Galpérine.

Conclusion. Les résultats démontrent d`après P. Ya. Galpérine d`une façon évidente la nécessité de particulariser le sujet du domaine étudié dès les premiers pas de l`éducation scolaire ce qui illustre une fois de plus le potentiel euristique de la théorie psychologique générale de P. Ya. Galpérine et la possibilité de l`appliquer aux différents domaines de la pratique sociale, principalement dans la résolution de problèmes éducatifs.

***Mots-clés*:** Théorie de l`activité de l`éducation; action; apprentissage; formation des notions; méthode formative; développement intellectuel; psychologie pédagogique; didactique; éducation

Points principaux:

- Les idées de Lev Vygotski sur le rapport entre l`éducation et le développement intellectuel et la théorie de la formation progressive des actions mentales de P. Ya. Galpérine agissent en tant que le fondement scientifique pour le développement des questions psychologiques liées au contenu et aux méthodes de l'enseignement scolaire.
- Programme d'auteur en mathématiques pour l'école primaire de Yakov Abramson, qui est basé sur la théorie de P.Ya. Galpérine, a montré une grande efficacité de formation dans le troisième type d'orientation et d'enseignement ; une telle formation contribue à la formation d'une pensée mathématique créative et au renforcement de l'intérêt cognitif.
- Au cours de la formation expérimentale, la thèse de P. Ya. Galpérine a été confirmée sur le lien entre les capacités intellectuelles des enfants et les types d'apprentissage : les élèves de première année sont capables de maîtriser les concepts mathématiques de base.
- L'analyse du programme confirme la position de P.Ya. Galpérine sur la nécessité de mettre en valeur une matière académique dès les premières étapes de son étude.

Introduction

Mathematical knowledge can be used skillfully and usefully only if it is mastered creatively.
A. Kolmogorov. "About the profession of mathematician"

Challenges in teaching and upbringing, as well as curriculum development and teaching methodologies for preschool education are interdisciplinary in nature, and therefore, are the subject of study not only in pedagogy, but also of related disciplines. The psychological aspects of the educational process have long attracted research attention that focused on establishing pedagogical conditions that not only support children' cognitive and personal development, but also fosters a capacity to adapt to rapidly changing living conditions. As A. Leontyev wrote half a century ago, "The education system today should shape the person of the third millennium. … current schoolchildren … in their future life will have to acquire much more knowledge than now, and besides, it will be knowledge of somewhat another type. Therefore, not instilling independence in them means dooming them to dramatic conflicts" (Leontyev, 2009, p. 410). This perspective is not limited to psychologists, for example, the famous scientist I. Feigenberg argues that teaching students to solve standard problems is insufficient; it is extremely important to "teach to continuously learn, to quickly find new necessary knowledge as soon as newly emerging problems reveal the need for it" (Feigenberg, 2008, p. 145). The prominent Russian mathematician V. Arnold emphasized the importance of mathematics in education, stating: "The tendency to expel all proofs from school education is especially dangerous. The role of proofs in mathematics is similar to the role of spelling and even calligraphy in poetry. Anyone who has not learned the art of proof at school is unable to distinguish correct reasoning from incorrect reasoning. Such people are easy to manipulate…" (Gubarev, 2000, p. 2). This highlights to the importance of focusing attention on the initial stages of school education. According to the G. Asmolov's metaphorical expression, "Primary school is the zone of proximal development of Russian society, where such words as 'innovative society' and 'creative society' should cease to be declarations, but should become deeds" (Asmolov, 2012, p. 316). Moreover, to paraphrase L. Vygotsky, it can said that primary school will lead society's development rather than trail behind it".

The subject of the current discussion is the psychological analysis of an experimental primary school mathematics program, built on the basis of L. Vygotsky's ideas on the relationship between learning and mental development as well as P. Galperin's theory of the gradual formation of mental actions and concepts. After establishing this program, Ya. Abramson has worked and continues to integrate this unique curriculum and teaching methodology in various schools in Moscow. His work with children, particularly the school program called *An intellectual*, has been the longest running of his efforts. This publication continues the discussion of the Ya. Abramson program (Stepanova, 2015), which began in 2015 in the "Issues of Psychology" journal, alongside its author (Abramson, 2015). The main problem was formulated by Ya. Abramson as follows: "… to teach via going **in depth** — *to solve school contest problems as well as the problems of increased difficulty, but within the framework of* *the national school curriculum in mathematics*, or via going **in breadth** - *to teach modern concepts that are not included in today's school curriculum, but allow a better understanding of the elementary mathematics' content as well?"* (Abramson, 2015, p. 59). According to the opinion of the program creator r, the combination of these two identified tasks when teaching school mathematics is necessary.

The psychological analysis of various training programs has long been a subject of researcher. A recent comparative analysis of learning outcomes in various primary school mathematics programs was recently carried out by A. Sidneva and colleagues as reflected in their publications (Sidneva, 2022; Sidneva, Aslanova, Bukhalenkova, 2022). The authors examined traditional educational programs and learning kits including *School of Russia*, *Planet of Knowledge*', *Perspective*, and *Primary School of the 21st Century*. They also explored programs with increased difficulty levels (including the mathematics teaching program developed by I. Arginskaya within L. Zankov's didactic system, the mathematics teaching program developed by L.Peterson, and the *Effective primary school* learning kit as well as the developmental education programs of the Elkonin-Davydov system. They concluded that the effectiveness of each program differed in terms of the mastering both general and specific mathematical skills. The materials presented below can be considered a continuation of the discussion initiated by the aforementioned authors regarding the advantages and disadvantages of various mathematics teaching programs, viewed through the prism of the key educational psychology issue: the relationship between learning and mental development. Mathematicians have also highlighted the fundamental differences in the results of mastering mathematics depending on how the learning process is organized As Kolmogorov (1973) stated, "A mathematics teacher in both higher and secondary schools is required not only to have a solid knowledge of the science he/she teaches. Only a person who is passionate about it and perceives it as a living, developing science can teach mathematics well" (p. 12).

Management of the Learning Process

Ya. Abramson mathematics program has been referenced in numerous publications (Abramson, 1993, 2012a, 2014, 2015a). Summarizing the experience of working in primary schools, Ya. Abramson observed that there are many children of primary school age, especially those who attended developmental preschool groups and received wide-ranging training at home who, when entering school, lack cognitive capacities, experience form of intellectual hunger. In particular, the analysis of school teaching practices led to Ya. Abramson to conclude that primary schools often do not meet the capabilities, demands and needs of preschoolers entering school (see, for example, Abramson, 1993). A paradoxical situation arises when children, already at the very first stages of schooling, lose interest in the acquisition of knowledge. This prompted Ya. Abramson to create his original program in mathematics, designed to accommodate the entire period of schooling. To date, this program for primary schools has been thoroughly developed and tested at the school named *An intellectual* (Abramson, 2012a, 2014, 2015a). In addition, manuals for teachers working schoolchildren from first to third-grade were published (Abramson, 2012, 2015, 2018; Abramson, Berezkina, 2013).

It should be specially noted that the teaching mathematics has long been a focus research in Russian psychology. In particular, under the leadership of P. Galperin, numerous studies were conducted by him, his students and followers, analyzing various aspects of mastering mathematical concepts (Butkin, 1968; Volovich, 1967; Galperin, 1965, 1985; Galperin, Georgiev, 1961[1]; Galperin, Talyzina, 1957; Davydov, 1958, 1966, 1969; Salmina, 1968; Salmina, Sokhina, 1968; Sokhina, 1968; Talyzina, 1955, 1995, etc.). Research results were subsequently analyzed by teachers to improve curriculum design and teaching practices. Furthermore, it is important to highlight the attempts made todevelop educational curriculum based on P. Galperin's theory of the gradual formation of mental actions and concepts has been (see about this: Galperin, Talyzina, 1979; Talyzina, 1992). Recently, the results of a longitudinal experiment conducted from 1965-1970 were published, focusing on the formulation of primary school mathematical concepts. This research was carried out under the guidance and with the direct participation of P. Galperin (Salmina, 2012). In summarizing the resulting data, N. Salmina noted: "The experiment on the formation of initial mathematical concepts showed the possibility of using the theory of the gradual formation of mental actions and concepts in mass education. This experiment also highlighted a number of problems that became the focus of further research, but did not lead to active discussion and analysis from the point of view of the peculiarities of organizing students' activities based on the theory of the gradual formation of mental actions and concepts in the context of mass education (Salmina, 2012, p. 103).

Study results on the peculiarities of mastering mathematical concepts in primary and secondary schools are detailed in the works of N. Talyzina (1988, 1995, 1998). The reason for the difficulties experienced by schoolchildren in studying mathematics, as well as other academic subjects, according to N. Talyzina, can be found in the psychological foundation on which the educational process is based. In Summarizing her many years of research, she draws a fundamental conclusion: organizing training based on the activity theory of learning, outlined by P. Galperin, enables all primary and secondary school students to successfully master mathematics. Talyzina (1995) notes that following the precepts of this theory of mastering can allow for ***control*** *of* the mastering process. (. a concept previously highlighted by P. Galperin. He emphasized that by organizing the transfer of external action to the internal mental plane, it becomes the subject's own property, asserting that "we control this process" (Galperin, 2002, p. 216). Thus, the widely held notion that school success is determined solely by children's abilities has not been confirmed.

Regrettably however, it must be acknowledged that these scientific developments have not influenced school practice. According to A. Podolsky, this can be explained by the fact that "the theory of the gradual formation of mental actions and concepts itself is not and has never been a theory, much less a teaching technology" (Podolsky, 2002, p. 25). Perhaps the most important condition for the effective practical application of the P. Galperin's theory is "not the desire for a literal reproduction of some abstract-universal procedure, but a creative psychological modeling of a specific situation (of learning) …" (Podolsky. 2023, p. 28). And, only in this latter case does the

[1] It is interesting to highlight: P. Galperin himself noted that in Georgiev's studies the interconnection between learning and mental development was especially clear (Galperin, Elkonin, 2001).

application of the P. Galperin's approach provide positive results. Furthermore, the educational system itself, including learning objectives, curriculum, teaching methods, means, and evaluation of results, takes on a fundamentally different form when compared to traditional schooling.

Theoretical and Practical Foundations of the Experimental Program

Before discussing the curriculum of the mathematics training program in question, it is necessary to delve into scientific and psychological principles. Additionally, test results of this experimental program have already been reflected in previous Abramson publications (see, for example, Abramson, 1993, 2012a, 2015a, etc.). Moreover, psychological analysis of the Abramson's program was the subject of our previous studies (Stepanova, 2015, 2023).

Methodology

Abramson's teaching methodology derived from L. Vygotsky's cultural-historical psychology, specifically, his exploration of the relationship between learning and mental development, which he identified as the central challenge of educational psychology.

L. Vygotsky approach to addressing relationship between learning and mental development from a historical perspective: he analyzed three possible existing methods to its solution proposing his own novel approach, which was fundamentally different from previously established methods. According to the L. Vygotsky's hypothesis, learning and mental development are neither two independent processes nor the same process; rather, there exists a "*unity*, but not identity, of learning processes and internal development processes [emphasis added]" (Vygotsky, 1991, p. 389). Acquiring knowledge from a human anatomy or progressing "forward in abstract thinking" are distinct processes. Vygotsky argued that there are complex relationships between learning and development, which became the subject of his special research. Examples of this research include the relationship between a child's oral and written speech, and the connection between everyday and scientific concepts. L. Vygotsky wrote, "Training is… an internally necessary and universal moment in the process of development in a child of not natural, but historical characteristics of a person" (Vygotsky, 1991, p. 388).

L. Vygotsky's experiments demonstrated, using the example of mastering written speech and grammar, the leading role of learning in the child's mental development. In his opinion, "learning is only good when it goes ahead of development" (Vygotsky, 1982, p. 252). Concurrently, development "is not limited to the 'more-less' scheme, but is characterized primarily by the qualitative new formations'" (Vygotsky, 1984, p. 247). It is no coincidence that L. Vygotsky called the principle of development the *metamorphoses principle* (Vygotsky, 2001, p. 22) given that development is not reduced to quantitative changes, but rather a qualitative transformation of one form into another. Vygotsky asserted that the child's development can be likened to the transformation of a caterpillar into a chrysalis, and a chrysalis into a butterfly.

With his concept development established L. Vygotsky then drew an important conclusion regarding learning. "Learning would be completely unnecessary … if it was not itself a source of development, a source of the emergence of something new" (Vygotsky, 1982, p. 252). When learning supersedes the early developmental stages, it awakens a whole series of functions that are indicative of the maturation stage. This concept was embraced by Russian researchers, confirmed through numerous experimental and theoretical studies, and contributed to the restructuring of school training practices, as reflected in the development of the Federal State Educational Standards.

L. Vygotsky concepts included "level of actual development", which is determined by the problems a child can solve independently and reflects the degree of maturity of their mental functions, and the "zone of proximal development," which is reached when a child solves problems in collaboration with an adult. "Research shows that *the zone of proximal development has more immediate importance for the intellectual development dynamics and educational success than the actual level of their development*" (Vygotsky, 1982, p. 247). Moreover, L. Vygotsky formulated a theoretically and practically important conclusion: "… the learning's optimal timelines for both the mass audience and for each individual child are established at each age by the zone of proximal development" (Vygotsky, 1984, p. 267). However, in light of the challenges discussed, we would like to draw attention to the thought of P. Galperin and D. Elkonin regarding the insufficiency of the term "zone of proximal development" as the concept of a zone proximity fails to address how learning occurs nor how learning effects development (ie. *thinking)*, or if any such correlation exists (Galperin, Elkonin, 2001, p. 310).

At school age, learning and development are related to each other as the zone of proximal development and the level of actual development. According to L. Vygotsky, the possibility of a child's transition from what they can do independently to what they can do in collaboration is the most sensitive indicator of successful childhood mental activity and is correlated with their zone of proximal development. For Vygotsky an important area of study concerning the challenges of learning and development would illustrate the relationship between spontaneous (everyday) and non-spontaneous (scientific) concepts. He wrote: "In essence, the problem … of scientific concepts is a problem of learning and development, for spontaneous concepts [including scientific concepts] make possible the very fact of their emergence from training, which is the source of their development" (Vygotsky, 1982, p. 224).

Thus, L. Vygotsky proposed a novel hypothesis concerning patterns of childhood psychological development and the role of learning in development. He articulated an important practical issue regarding the school educational curriculum, which aims at mastering a system of scientific concepts. However, the question of how this mastering should be organized, and what conditions would promote or hinder its successful progress, remained open. Additionally, the L. Vygotsky's thesis itself "learning leads the development" discreetly implies the need for such a special organization of mastering.

In light of the problem under discussion, P. Galperin's reference to L. Vygotsky's experimental study of the concepts' development in a child is of particular interest. P. Galperin emphasized that when concepts are formed using the method of gradual formation of mental actions, neither complexes, pseudo-concepts, nor intermediate forms arise from elements of everyday and scientific concepts. "… a child can neither miss any essential feature of a concept nor introduce into it something unessential" (Galperin, 1965, p. 22). The reason for the difference between these data and the well-known data by L. Vygotsky was explained by P. Galperin through the peculiarities of the experimental research procedure: while L. Vygotsky studied the possibilities of the independent formation of concepts, P. Galperin investigated the possibility of teaching children this action through gradual formation. Therefore, P. Galperin made an extremely important practical conclusion, which not only has not lost its relevance, but has also acquired an even more modern sound. He argued: "… the course of the concepts' formation in modern school training, which… controls the process only by the *final result*, cannot be considered as *mental development normative standards and learning's natural boundaries* [emphasis added]" (Galperin, 1965, p. 23).

The Theoretical Basis

Ya. Abramson 's program in mathematics was informed by P. Galperin's theory, and above all, his guidance concerning the three types of orientation and their connection with mental development (the three types of orientation shall be outlined in detail below). Although P. Galperin was neither an educational psychologist nor a teacher; he observed general psychological theory as he developed educational practice. It is likely no coincidence that the last *lifetime conference* dedicated to P. Galperin's 80th anniversary had the following title: *The significance of the P. Galperin's theory of the gradual formation of mental actions for the psychological science development and teaching practices' improvement* (Stepanova, 2017). P. Galperin emphasized that due to the structuring of childhood learning activities, "a new solution to an old problem was obtained - to the problem of academic performance's equalization within a sufficient mass of people" (Galperin, 2002, p. 217). When using the gradual formation method, a general educational program becomes accessible to all children. P. Galperin wrote: "… there is an equalization of academic performance. I want to emphasize: of performance, not of ability. … we do not deny them, and they also must have their own explanation. But in relation to the general educationaл program, you can get academic performance's equalization" (Galperin, 2002, p. 218). He specifically highlighted: "… there must be a different approach to mass education — through *a change in teaching methods [emphasis added]*" (Galperin, 2002, p. 251).

Moreover as V. Zinchenko asserted, "biographically, meaningfully and spiritually P. Galperin was associated with the L. Vygotsky's school… But while preserving this cultural and historical code, we should not forget that P. Galperin owns his own body of ideas, on the basis of which his own scientific school later developed" (Zinchenko, 2002, p. 120). If L. Vygotsky's main question asked *what to teach*, then P. Galperin focused on *what and how to teach*. P. Galperin, on the one hand, spoke about the need to transfer generalized knowledge to the child, and, on the other, focused on the optimal teaching methods for development. Furthermore, the question of *what to teach* correlates with the question of *how to teach*. It was these questions that was reflected in the last lifetime publication by P. Galperin "Teaching methods and child's mental development" (Galperin, 1985).

P. Galperin's position can be illustrated in what follows. In traditional teaching, knowledge acquisition occurs without sufficient guidance from the teacher and is regulated by assessment results. P. Galperin considered *action* as a subject to be mastered and set the following task: to determine the conditions "taking into account of which not only ensures, but, one might say, even forces the student to act correctly and only correctly" (Galperin, 1998, p. 428). This system is called *the gradual formation of mental actions and concepts* and it includes four groups of conditions (Galperin, 1985, p. 4):

1. Establishing adequate student motivation.
2. Ensuring the correct execution of action.
3. Nurturing its desired properties.
4. Transforming it into mental action (its desired form).

With such a structured approach to learning, a new action is formed more easily and quickly than traditional school education. This is ensured primarily because the trial-and-error processes in the initial stages of action's formation is eliminated. The advantages of the P. Galperin's teaching method, which "allowed the child, after analyzing the first few objects, to independently explore any new object of the same kind, establish its structure and its features, independently reproduce it based on these features, and in action assimilate both knowledge of the object and actions with it" (Galperin , 1985, p. 30), were demonstrated by the theory's author, and subsequently by his students, on various educational materials. This is especially important for us as it pertains to, *elementary mathematical concepts*.

As Ya. Abramson noted, the differences between this teaching method and the traditional formation of concepts are striking. P. Galperin provides a detailed description of types of learning, with each distinguished by "its own orientation in the domain, its course of the learning process, its results' quality and the children's attitude to the process and subject of learning" (Galperin, 1985, p. 30). In the *first teaching type*, the orienting basis of an action (OBA) is incomplete, resulting in numerous trials and errors. In this case, the formation of the action occurs by controlling the final result, which does not ensure the correlation of conditions with specific operations. Therefore, the action is very unstable in response to changes in environmental conditions and is limited in its transfer to new tasks.

The *second teaching type* is characterized by the construction of a specific action based on a complete orienting basis, where trials and errors become random and non-characteristic. Each operation is clearly correlated with the conditions and, as a result, the action is reasonable (within specific conditions), generalized (to a predetermined extent) and conscious. The formed action is resistant to changing conditions, with significant transferability, though it depends on the degree of identity of the elements that constitute the action.

The *third teaching type* involves orientation on the material's basic units, the laws of their combination, methods for determining both, and the independent construction of OBA for specific objects. a complete OBA ensures the formation of actions and concepts without trials and errors. Actions become reasonable not only in terms of their correlation with conditions, but also the conditions themselves, revealed in their internal structure. The action is fully and widely transferable. This teaching approach fosters not only a conscious attitude towards one's actions, but also "higher consciousness from the point of view of the objective position and movement of the things themselves" (Galperin, 1998, p. 311). According to P. Galperin, mastery of an objective criterion and especially the transition "to the position of such a proper mediated approach" (ibid.) to reality produces "a revolution in the child's thinking" (Galperin, 1998, p. 311).

Teaching types differ not only in accordance with the subject's composition, but also in the manner of presentation by the teacher and the construction of orientation by the student. Different teaching types naturally have a range of effects on mental development. When training according to the first type, there is an accumulation of narrow set of knowledge and skills, thinking and ability development occur in isolation from the training; the second teaching type also lacks an impact on mental development, due to an applied interest in knowledge rather than a theoretical or cognitive emphasis. And only the third teaching type presents a fundamentally different picture most relevant to research. "…The first and most important thing in the third teaching type is the cognitive activity stimulation, the increasing strengthening and development of cognitive interest itself" (Galperin, 1985, p. 34), which determines the powerful developmental effect of learning.

With respect to the third teaching type, P. Galperin observes two foundations: the method of studying objects and the method of involving students in this research, which are inextricably linked. He states, "the focus on studying an object… places the main goal of activity in the very process of cognition… arouses the cognitive interest itself… and encourages the use of those opportunities that the research method opens up" (Galperin, 1985, p. 36).

Based on the conducted research, P. Galperin made an important conclusion regarding the age-related possibilities of mastering scientific concepts: already at the senior preschool age, using the method of gradual formation of mental actions and concepts, it is possible to form full-fledged concepts "which were ahead of the rest of the consciousness's content for the entire period of mental development" (Galperin, Elkonin, 2001, p. 311).

Ya. Abramson 's program in mathematics, which will be discussed further, is built primarily on the principles of the third teaching type, based on cognitive interest, which, as noted by P. Galperin, requires the exclusion of other motivation types. Of note, A. Kolmogorov also drew attention to the need for designing mathematics teaching on the basis of "pure curiosity" (Kolmogorov, 2001b, p. 190), since "mathematics can be interesting to everyone and useful everywhere" (ibid.).

Furthermore, even a quick glance at Ya. Abramson's program in mathematics reveals some elements indicative of the first and second teaching types. This may be due to the characteristics of the children who studied at the school named *An intellectual*: the program was designed for gifted students, yet in the same class, there are often children with varying levels of mental development and different levels of preparedness for schooling.

The program's *practical foundations* were based on educational and methodological requirements developed by Ya. Abramson, which ensured both motivational readiness to learn new things and high intellectual activity in the classroom.

Firstly, the fast pace of training sessions, characterized by frequent changes in tasks and limited repetition, helps prevent fatigue by encouraging the search for non-standard, novel solutions.

Secondly, the gradual work with actions, incorporating them in all subsequent types of tasks, for example, mastering columnar addition, children can move on to multiplication, which draws upon the action of addition.

Thirdly, teaching children whose mental development level exceeds that of their peers, creates conditions for success in school contests, which then increases teacher motivation. This ultimately ensures the successful participation of all individuals in the educational process - both the students and the teachers.

Fourthly, the continuity of classes, where homework is assigned during the holidays, ensures both intellectual endurance and an insatiable intellectual need given that children enjoy solving difficult problems. Homework for the summer holidays differs significantly in volume as it is approximately ten times larger than typical assignments given for a weeks-long vacation during the school year. In addition, its curriculum exceeds previously learned material with new information added, which the student must master independently. All summer assignments are subsequently assessed and discussed in class, and students are very happy to share their solutions with classmates.

Fifthly, presenting a general scheme for analyzing the problem's statement with limited explanations, encourages intellectual struggle or *torment* and preserves the children's creative approach to learning, which frees them from the limited reliance on simply solving problems according to *example*.

Sixthly, consistent with P. Galperin's ideas concerning failure, recognizing the learning value in a child's making mistakes, serves as the basis of subsequent joint discussion and analysis, which rather than weaken the child's interest in the task inspires curiosity and a search for new non-standard solutions.

Results and Discussion

An Experimental Program for Teaching Mathematics through the P. Galperin's Theory Prism

Based on the presented above, significant psychologically and pedagogically aspects of Ya. Abramson's mathematics program can be highlighted.

First of all, it is important to highlight the relevance of the issue concerning the content and teaching methods of school curricula - both in primary and secondary levels. This relevance is reflected in its active discussion in periodicals, at congresses and conferences, as well as through the emergence of various professional development courses for teachers.

Whom to Teach?

This program was initially piloted with children attending the *Aleph* private school. The class consisted of 8-10 students, typically including 1-2 with fairly high (above average) mental development levels, and 2-3 student with developmental disorders of varying severity. The mathematics program was successfully mastered by 75–80% of children.

Subsequently, based on this experimental program, training in mathematics was introduced at the school named *An intellectual*, which involved competitive selection. Enrollment in the school was based on interviews results, during which children were asked to solve a number of problems. At the same time, the main focus was paid on whether the child solved the problem (as the problems were of increased difficulty), but rather to determine how long they worked on solving it independently without asking adults for help. The program's author wrote: "The main thing is to have the ability to focus on a task, to fix your attention for a long time" (Abramson, 2012, p. 11). It is interesting to note that a similar position is shared by L. Peterson, the author of mathematics textbooks for primary schools, who developed a school mathematics curriculum aligned with the principles of developmental education. In her opinion, "in primary school the greatest difficulties are experienced not by those children who have not received this or that amount of knowledge, but by those who show intellectual passivity, who lack the desire and habit of thinking, the desire to learn something new" (Peterson, 1998, p. 51). As a side note, the intellectual passivity phenomenon was studied by L. Slavina in the L. Bozhovich laboratory (who was a student of L. Vygotsky, and, like P. Galperin, in her research she relied on her teacher's ideas about the child's mental development patterns).

Another important criterion in assessing the child's suitability was their ability to listen to both teachers and classmates, without being distracted by questions unrelated to the task.

Also, intelligence and reaction speed were taken into account, which became apparent when solving *riddle tasks*.

However, it should be noted that, in Ya. Abramson opinion, this program is suitable for teaching all children which is consistent with evidence that indicates learning will occur at varying speeds.

Why to Teach?

The program's developer, Ya. Abramson, set himself the goal of introducing children to "real mathematics, similar to what they will have to do if they decide to choose it as a profession, and so that they make this choice consciously" (Abramson, 2012, p. 6). This goal emerged from years of personal study in mathematics school, during which the author developed a critical perspective towards mathematics as merely a set of school contest problems. Consequently, the training's purpose was "to instill in children a love of mathematics, to form an understanding of this discipline, and to develop 'mathematical thinking'" (Abramson, 2012, p. 7). The task posed in this way guided the author's search not only in selecting the curriculum of the material to be mastered, but also the methods of its sequential assimilation.

How to teach?

Ya. Abramson's objective was to develop in children the ability to independently derive rules and theorems, and not to work according to a predetermined algorithm.

This is precisely the approach that was implemented in studies conducted by P. Galperin. As mentioned above, the mathematics program was designed in accordance with the requirements of the third teaching type, and therefore the following features are reflected in its design including:

- The presence of cognitive motivation prior to starting school and throughout the process of learning mathematics. Additionally, a negative attitude towards the academic subject is mitigated by the issuance of stars for the correct solution, while erroneous answers are recognized without criticism.
- Creating a complete and generalized orientation that ensures the action's correct execution, while also fostering a creative attitude among younger schoolchildren toward tasks that are new to them. A clear illustration of this is children's independent derivation of proofs and rules. While memorizing mathematical information has a logical outcome, it is never the goal of the training.
- Gradual transfer of the action's orienting part into the mental plane due to through the obligatory varbalization of the operations being performed.
- Concern for nurturing the action with given properties, with particular attention paid to fostering consciousness as one of the key properties.

What to Teach?

According to Ya. Abramson, the program's author, the main question that confronted him was whether to focus efforts on mastering increasingly difficult mathematical problems or to prioritize the mastery of general concepts and the demonstration of connections between various tasks. There is strong evidence to suggest that, by prioritizing the mastery of a system of mathematical concepts from the earliest stages of education, the author unwittingly ensured that children turned out to be very successful in solving complex non-standard problems. Moreover, this approach specifically focused on mathematics as an independent and integral field of scientific knowledge, rather than the traditional division into algebra and geometry that is currently prevalent in school education.

The program's curriculum is presented through oral and written presentations by its author and is mainly methodological in nature, intended for mathematics teachers. It is important to note that familiarization with the program requires a relatively high level of mathematical literacy as the changes concern not only the content of familiar topics within elementary school curriculum, but also the order in which they are taught. Additionally, new topics were introduced that were not previously included in the curriculum. In other words, working with this program cannot be reduced to just changing some methodological techniques but rather requires a fundamentally different approach to introducing children to mathematics, in contrast to traditional curriculum. As a result, discussing the outcomes of implementing the program with primary school teachers appears to be a highly promising avenue for further exploration.

Conclusion

According to Ya. Abramson, mathematics lessons structured according to his design conclusively demonstrates that children can derive pleasure from their studies, and fully master the curriculum of instruction, which significantly exceeds that of traditional school. This speaks in favor of the increased developmental capabilities of children at these ages. The high praise of this training program form parents is also noteworthy as they report not only the children's positive attitude towards the learning process, but also their lack of fatigue after school classes.

During the experimental teaching of mathematics, it was possible to show that the widely held opinion concerning the age limits of primary school students is misleading. In particular, as a result of the program's design, first-graders are able to use calculus systems, carry out operations such as exponentiation, taking roots and calculating logarithms, and master basic geometric concepts (e.g. theorem, axiom, methods of proof and so on).

Throughout the entire study period, there was a sustained interest in the subject, evident from the fact that the children continued to solve problems during their breaks and asked for more homework. According to Ya. Abramson, fast pace, relatively high difficulty of tasks, and their infrequent repetition contribute to maintaining cognitive motivation in general, and specifically in relation to mathematics.

Moreover, Ya. Abramson believes that this new curriculum, which involves changing the material's content as well as its order, will significantly alleviate the burden on the middle and high school curriculum. It will even be possible to teach a higher mathematics course, equivalent to the first two years of university mathematics.

It is also essential to note the positive effect of training for a teacher working based on this program: "What happens is what ideally should happen in the classroom: emotional intensity, high intellectual tension, transmitted in both directions: from teacher to children, from children to teacher." (Abramson, Berezkina. 2013, p. 4).

As we showed earlier, "the main theoretical conclusion is related to the confirmation of the well-known position about the learning results' dependence on its content and methods; in particular, it was possible to once again show how effective is learning based on the third teaching type" (Stepanova, 2015, p. 78). The P. Galperin's thesis about the connection between the children's intellectual capabilities and teaching types, about the fallacy of considering capabilities in isolation from the teaching type was also confirmed further. In general, one cannot help but recognize L.Obukhova's words as prophetic : "… works by P. Galperin are marked by historical significance, and *psychology will sooner or later develop along the path outlined by him* [emphasis added]" (Obukhova, 2014, p. 74).

The age at which one can engage in in-depth study of mathematics deserves special attention. In this regard, the subject of a special study could be a comparative analysis of the experimental mathematics teaching results and data on age-related talent obtained by N. Leites. Data obtained in psychology on the presence of age-related sensitivity and selectivity of attention to certain influences allowed N. Leites (Leites, 2000) to conclude that extraordinary mental abilities depend on the childhood's specific capabilities. P. Galperin, touching on the the issue abilities wrote: "Does such a gradual mental activity formation deny the presence of abilities and differences in abilities? Of course not. It denies not the existence and difference of abilities, but the abuse of the abilities' concept to justify pedagogical defects and third-rate pedagogical products" (Galperin, 1960, p. 144). However, "in relation to the general education program, you can get equalization of academic performance… Here is a different approach to mass education, through a different teaching methodology" (Galperin, 2002, p. 218). In light of this issue, it should be emphasized that the mathematical abilities of children studying across different programs was explored in the research mentioned above (Sidneva et al., 2022).

In considering Ya. Abramson's program, it is important to highlight their intention to develop children's creative abilities. This clearly indicates the possibility of organizing training that fosters creative mathematical thinking. We emphasise this point because P. Galperin was well aware of objections raised by his opponents, which addressed in his doctorate thesis (Galperin, 1965), one of which argued that the third teaching type promotes the student's creative activity. The arguments presented by P. Galperin in favor of creative exploration in the third teaching type were clearly confirmed through practical application in this case.

The results of experimental mathematics teaching allowed Ya. Abramson, the program's author, to conclude: "… it is precisely the period during primary school, 7-11 years old, that is the most productive in terms of the possibilities for the formation of so-called "mathematical thinking", the preservation and identification of mathematical abilities that are present in a much larger number of children than is commonly believed and much more , than we see at the turn of 5th grade" (Abramson, 2012a, p. 41).

At the same time, the difficulties that have emerged in the implementation of this program should be noted. At the very least, it is short-sighted from a practical point of view to ignore the problems that may hinder its further progress. The difficulties we identified primarily include the following:

- One of the main challenges is related to teacher training: currently, the transfer of teaching experience occurs *face to face*, which, enabling an adequate understanding of the teaching methodology, it greatly complicates its application. This results in the difficulty of expanding its use further.
- The issue of organizing training under this program in a public school remains unresolved.
- The teaching of other academic subjects is also an open question, as well as the relationship of this program with other mathematics programs not only in primary, but also in middle and high school.
- It is unclear how to address underperforming children: if the mass school program is designed for all children with a normative development without exception, then what should be done in case of insufficient mastery of this material? Is it advisable to continue their education within this framework? This issue has become particularly acute with the increasing focus on inclusive education.

In conclusion, it should be noted that at the present stage this program is still experimental in nature and more time is needed before drawing conclusions regarding its effectiveness. However, one thing is certain: without its testing it, there will be insufficient information to make conclusions. Therefore, the purpose of this publication is modest: to draw attention of psychologists, educators, theorists and practitioners to one of the author's mathematics programs for primary schools. The discussion of this program will lead to a better understanding of the curriculum and methods of teaching mathematics in modern schools. It is also promising to consider the continuity between the mathematics curricula in secondary and higher education.

We once again reaffirm P. Galperin's statement about the need to highlight an educational subject from the very first steps of its study: "… just at the first approaches to science, at the first acquaintance and even the introductions to it, a clear identification of its subject is especially important and constitutes an imperceptible, but irreplaceable condition for its further study" (Galperin, 2023, p. 555). The general implication here concerns the significance of P. Galperin's theory with respect to current and future educational practice (Stepanova, 2023).

Furthermore, of independent interest is an appeal to the results obtained by Ya.Abramson, not only from psychological, but also from pedagogical persepctives. The renowned Russian mathematician A. Kolmogorov, also known for his work in teaching mathematics to schoolchildren, stated: "… as a practitioner, I am inclined to think that the nature of mathematical development, achieved according to the most modern recipes for early studies in set theory and algebra, up to 10-12 years of age with a fairly good success could be replaced with the general education of intelligence and mental activity. But the delay in mastering strict logic and special mathematical skills at the age of 14-16 becomes difficult to compensate for" (Kolmogorov, 2001a, p. 106). A comparison of these authors' approaches is a worthy subject for future research.

Conflict of Interest

The authors declare no conflict of interest.

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